Mathematics B-day 2014

Monday December 1, 9:00 - 16:00 hr



Lights Out



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Warming up: play the game!

The assignment is about the game 'Lights Out', which has been played worldwide since 1995. The game is played on a grid of five rows by five columns with a total of 25 lights. Some of the lights are on at the start of the game. The point of Lights Out, as can be seen from the name, is to switch off all the lights. You can click on lights (they are in fact buttons with a light in them). If you click on a light, you toggle the state (so if the light was on, it switches off; if it was off, it switches on). There is an important addition: if you toggle a light by clicking on it, you will also toggle its direct neighbours (left, right, top, bottom).

Today, you will find out that there is a lot of mathematical puzzling to do in Lights Out! You will also look into variants of the game that are played on different-sized grids.

First, as a warming up, here are a few Lights Out problems. Open the app via: <u>http://www.fisme.science.uu.nl/wisbdag/opdrachten/LightsoutEN/Lightsout.html</u> and take some time to explore the options in the app. Your teacher will have an offline version (Windows only) available.

Puzzle 1 (easy)

Create a 5×5 grid in the app, and use the option 'toggle lights individually' to make the start configuration as seen on the right. Now deselect the option 'toggle lights individually' and select the option 'Execute clicks immediately' under 'Play the game'. Solve the puzzle (i.e. turn off all the lights). A 0 means 'off', a 1 means 'on'. This puzzle can be solved in five clicks.

1	1	0	1	1
1	0	1	0	1
0	1	1	1	0
1	0	1	0	1
1	1	0	1	1

1	0	1	0	1
1	0	1	0	1
0	0	0	0	0
1	0	1	0	1
1	0	1	0	1

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Puzzle 2 (medium)

Puzzle 3 (*difficult*) Is this puzzle too difficult right now? No problem!

Solve the next puzzle (it can be done in six clicks).

After a lot of fruitless clicking you may even think: *it's impossible!* In this case you are wrong, because it is possible but we can already tell you now that only 25% of the 33 554 432 possible start configurations is solvable. Solvable means that there is a solution; it is possible to turn off all the lights. Unsolvable means that no matter what you try it is impossible to turn off all the lights.

Later in this day you will learn techniques that help you decide for a given puzzle *whether* it is solvable or not and *how* you can find the solution if it is solvable.

Practical points

The structure of the assignment and schedule for the day

In part ONE you will explore Lights Out, and try out various things yourself. Part ONE is primarily intended for the morning. There are quite a few assignments where you have to draw something or write something down. The assignments in this part are numbered. Later (for instance, in part TWO) you can refer to something you did before. For example: "In question 13 we saw that We are using that again now."

The exploratory questions, which are called **[Exploration]**, are meant to introduce something new, and these questions are not included in your final report. For the other questions, which are called **[Assignment]**, you have to include an answer with correct reasoning in your final report.

Warning! The final part (E) of part ONE is rather difficult and theoretical. Only work on it if you still have enough time in the morning. This part is specifically important for answering Main question D in part TWO.

In part TWO (own investigation) you will work on more complex and open assignments. Part TWO is primarily intended for the afternoon. In this part you can apply what you learned in part ONE. You will work on one or more of the Main assignments A, B, and C. Additionally, if you want you can also work on Main assignment D. Keep in mind that answering two Main assignments in depth will be appreciated more than superficially treating three or four Main assignments in part TWO. The answers to these Main assignments are to be included in your final report.

What do we expect from you?

In your *final report* you will describe your results from the *assignments* in part ONE and the Main assignments in part TWO.

Present your report so that it is clear and convincing. Of course you should include relevant explanatory figures as illustrations. Make sure that your report is understandable for people who did not take part in the Mathematics B-day, but who do have a sufficient mathematical background. This means that you must introduce the problems clearly and that, where relevant, you will use your explorations and arguments from part ONE.

Summarising: you will write a clear report, supported by mathematical arguments. The quality of the report will play a part in assessment!

For the final report it may be useful to keep of what you do in the morning in a digital document. Keep in mind that you must be able to hand in the completed final report by four o'clock in the afternoon!

Information on the Internet

You can find a lot of information about playing Lights Out on the internet, but this information is more likely to confuse you than help you with the problems you will work on today.

Good luck, but above all: have fun with this mathematical challenge!

Part ONE

You indicate the **position** of a light on the grid with (*row*, *column*). A click on light (*i*, *j*) is written down as $K_{i,j}$. If there is only one row, you click on light *j* with K_j . A *click series* is performing multiple clicks, for example $K_{1,3}$ $K_{4,2}$ $K_{3,3}$. The *null configuration* is the configuration in which all lights are off (i.e. 0).

A: Click series

1. [Exploration]

Make this configuration on a 4×7 grid by using the app (use 'Configure the grid' and 'Make a configuration'):

1	1	0	0	1	1	0
0	0	0	0	1	1	0
0	1	0	1	1	0	0
0	0	0	0	0	1	0

The underlined numbers indicate which lights will be clicked (you can do this yourself by selecting the option 'make a plan' in the app):

1	1	0	0	1	1	0
0	0	0	0	1	1	0
0	1	0	1	1	0	0
0	0	0	0	0	1	0

Before you click on 'execute plan' you can already figure out that light (2,3) will not be toggled and light (1, 5) will be.

- a) Explain how that is related to the number of clicked neighbours.
- b) On a 1×99 grid, all the lights are on. You perform the click series $K_1 K_2 K_{50} K_{92} K_{94}$. Which lights are now off?

2. [Exploration]

Make a random configuration on a 1×6 grid, and write down this start configuration.

- a) Perform the click series $K_1 K_3 K_4 K_5 K_3 K_5$ and write down the resulting configuration.
- b) Return the grid to the start configuration and perform the click series $K_1 K_4$. Write down the resulting configuration.
- c) The final configurations in **questions a) and b)** turn out to be identical. Explain why that is the case.

3. [Exploration]

Make a configuration on a grid of your choice, and write down this start configuration Make a click series of length three; call these clicks *A*, *B* and *C*. Verify that *A B C* has the same effect as *B A C* and as *C A B*. Also explain why this is the case.

Summary

- You can remove double clicks from a click series. A click only matters if it occurs an odd number of times.
- The order of clicks in a click series is unimportant.

The above has important consequences that will be of constant relevance today.

Consequence 1: You can always go back.

4. [Exploration]

If you perform the same clicks series twice, you will be doing every click two times. So, in one word, what happens?

5. [Exploration]

If from a specific start configuration you have found a click series that takes you to the null configuration and you repeat that same click series, what will be the final configuration?

Consequence 2: The solvable configurations are precisely those that you can make by clicking starting from the null configuration.

6. [Exploration]

Explain why consequence 2 has to be valid.

7. [Exploration]

- a) Use the app to make a 3×4 grid. Click the button 'all lights on'. Click the button 'make a plan' and click on all the lights along the edge of the grid, including the corners. Do *not yet* click the 'execute the plan' button. Predict what will happen when you execute this click series.
- b) Click the button 'execute the plan'. Does the result match what you predicted?
- c) What will happen if you click on all the lights along the edge of the grid again?
- d) In the same way, investigate what happens if, on an $n \times n$ grid, you use all the lights along the diagonal ((1, 1), (2, 2), (3, 3) up to and including (n, n)) as a click series.
- e) First toggle all the lights off using the button 'all lights off'. Now, click on every light once. Predict the result for *m*×*n* grids for some values of *m* and *n*.

8. [Assignment A]

We will now look at a $1 \times n$ grid (for n = 1, 2, 3, 4, ...) with all lights turned on in the start configuration.

Claim: There is a click series for every value of n to reach the null configuration.

- a) Show that this statement is true for all values of *n*.
- b) For every value of *n* there is also a shortest click series to reach the null configuration. Investigate the relationship between the length of that click series and *n*.

B: Working with zeroes and ones and toggle diagrams

A click on a light will toggle the state of that light and its direct neighbours. In other words, this changes the configuration on the grid.

Here is an example on a 4×5 grid. You can represent the result at click $K_{3,3}$ like this:

c	Start configuration					click K _{3,3}						Final configuration				1
1	1	0	1	0		0	0	0	0	0		1	1	0	1	0
1	0	0	0	1		0	0	1	0	0	_	1	0	1	0	1
0	0	1	1	0	+	0	1	1	1	0	-	0	1	0	0	0
0	1	1	0	1		0	0	1	0	0		0	1	0	0	1

The ones on the grid in the start and final configuration are the burning lights. The ones at click $K_{3,3}$ represent something else: namely that those five lights will be toggled by the click (from 'on' to 'off' or the other way around). Here, the zeroes represent that these lights are not being toggled.

We call the 4×5 rectangle under $K_{3,3}$ the toggle diagram of $K_{3,3}$

The '+' and '=' in the figure indicate that you can 'calculate' the final configuration. You need to apply the following 'calculation rules' in each cell:

- 0 + 0 = 0 a light that is off and is not toggled stays off,
- 1 + 0 = 1 a light that is on and is not toggled stays on,

0 + 1 = 1 a light that is off and is toggled is turned on,

1 + 1 = 0 a light that is on and is toggled is turned off.

You can represent the effect of two subsequent clicks (first $K_{3,3}$ and then $K_{2,4}$) as follows:

Ċ	confi	Star igur		n			cli	ck k	3,3			click K _{2,4}					Final configuration			า		
1	1	0	1	0		0	0	0	0	0		0	0	0	1	0		1	1	0	0	0
1	0	0	0	1	+	0	0	1	0	0	1	0	0	1	1	1	=	1	0	0	1	0
0	0	1	1	0	т	0	1	1	1	0	+	0	0	0	1	0	-	0	1	0	1	0
0	1	1	0	1		0	0	1	0	0		0	0	0	0	0		0	1	0	0	1

You can also first calculate the result of the two clicks together and then determine the effect of the click series ($K_{3,3} K_{2,4}$):

C	Start $K_{3,3} + K_{2,4}$ configuration							Final configuratio				า				
1	1	0	1	0		0	0	0	1	0		1	1	0	0	0
1	0	0	0	1		0	0	0	1	1	_	1	0	0	1	0
0	0	1	1	0	+	0	1	1	0	0	-	0	1	0	1	0
0	1	1	0	1		0	0	1	0	0		0	1	0	0	1

The rectangle under $K_{3,3}$ + $K_{2,4}$ is called the toggle diagram of click series $K_{3,3}$ $K_{2,4}$

Pay attention to the difference between click series and toggle diagram!

Summary

- a click series shows which lights are clicked on;
- the toggle diagram of the click series shows which lights are toggled.

9. [Exploration]

Take a grid, for example 7×4, and think of a click series of 3 clicks. You can find the toggle diagram for this click series like this: Turn off all the lights and perform your click series. Done! Now explain why you SEE the toggle diagram of the click series on the screen.

10. [Exploration]

On a 7×7 grid, what does the toggle diagram look like if all the lights of the British flag are clicked: all lights on the two diagonals and all lights in row 4 and in column 4.

11. [Exploration]

Quite a number of *really different* click series can be performed on a given grid (*really different* meaning: every light is clicked on no more than once and the order is not relevant).

- a) How many really different click series exist on a 1×4 grid?
- b) How many really different click series exist on a 2×2 grid?
- c) Find click series for the following four toggle diagrams (on 1×4 and on 2×2):



12. [Assignment B]

Consider the 1×5 grid. Thinking and reasoning is enough for this assignment; trying things out in the app will be very helpful.

In **question 8** you found that from the null configuration of the 1×5 grid you can switch on all the lights with just two clicks, and, of course, switch everything off as well. $K_1 K_4$ was one of those click series.

 K_1 toggles two lights, K_4 the other three. The toggle diagram has five ones. But there is a different way ...

a) The toggle diagram of click series $K_1 K_4$ is the same as that for $K_2 K_5$. Verify this.

We will call $K_1 K_4$ and $K_2 K_5$ parallel click series.

Of course, this is only significant if the click series themselves are different.

You might think that this is an exceptional situation. But it is not! In a few steps, you will now find out that there are many pairs of parallel click series on a 1×5 grid!

b) If you perform $K_1 K_4 K_2 K_5$ nothing happens (i.e., the configuration does not change) Why is this?

From now on we will call a click series that does nothing a *silent click series*.

- c) Explain why the click series $K_1 K_2$ and $K_4 K_5$ are parallel as well.
- d) Find a click series that is parallel with the click series that only consists of K_1 .

In two parallel click series, there can be clicks that occur in both series. This is something that you will need now.

e) Find a click series that is parallel with $K_1 K_2 K_3$

Using the following 'recipe', you are able to find parallel click series: After a click series you selected yourself, append $K_1 K_2 K_4 K_5$ to the series and remove the double clicks.

The click series you end up with is parallel with the one you started with.

- f) Check that this recipe works for the examples already given, and give an explanation of why the recipe works.
- g) On the 1×5 grid there are 16 pairs of parallel click series. Why is this?

13. [Exploration]

- a) You can also make parallel click series on a 1×8 grid. Make a partner for $K_1 K_4 K_7$
- b) What is the smallest grid that has parallel click series?

14. [Exploration]

- a) Every grid has as many possible configurations as there are possible click series. Why is this?
- b) Sometimes there are not as many toggle diagrams as there are click series. Why is this?

Remember what we found in **question 6**:

The solvable configurations are precisely those that you can make by clicking, starting from the null configuration.

c) So, on some grids you cannot reach every configuration from the null configuration. Why is this?

Concluding, on some grids there are start configurations for which you can find no click series that results in the null configuration.

What a pity.... But at least now we know.

Now, let's look for the ones that can be solved and how they can be solved.

C: 1×n grids and 'mutually toggling' lights

In this part you will learn a technique that gets you pretty far on 1xn grids. It is an introduction, though. In Main assignment A in part TWO you can finish the work yourself.

15. [Exploration]

In this Exploration you will use a $1 \times n$ grid, with n > 3.

- a) Which toggle diagram goes with click series K₂ K₃?
 What does that mean for lights 1 and 4? And for lights 2 and 3?
- b) Which toggle diagram goes with click series $K_j K_{j+1}$? What does that mean for lights *j* - 1 and *j* +2?
- c) Which toggle diagram goes with click series K_1K_2 and with click series $K_{n-1}K_n$? Which lights will now be toggled?

This idea of clicking two 'neighbouring lights' can be applied to all $1 \times n$ grids.

16. [Exploration]

On this 1×15 grid the crosses mark the click series:



What toggle diagram goes with this click series?

Concluding, with this technique you can toggle lights have distance of 3 between them. By repeating the technique you can also bridge distances that are a multiple of 3. In the example the distance was 9.

17. [Assignment C]

As an example we will look at a 1×10 grid. Using this technique, the lights in the group {1, 4, 7, 10} can mutually toggle each other.

a) Which group of mutually toggling lights do you get with light 2? And with light 3?

Lights 1 and 2 also mutually toggle, as do 9 and 10.

- b) On a 1×10 grid the only lights that are on are 1, 3, 7 and 9. Find (use 'make a plan') a click series that lets you reach the null configuration.
- c) Now, the only lights that are on are 1, 3 and 8. What click series will let you reach the null configuration?

D: Chasing

A technique that is often mentioned online as a way to approach Lights Out puzzles is called *chasing*.

For a given start configuration you can switch off all lights that are on in all rows except the bottom row. This is how it works. For example, in row *i* several lights are on. Click on all the lights in row i+1 (the row below it) in those columns where there light is switched on in row *i*. Start on the top row and work your way down like this.

An example

Given the start configuration on the 4×3 grid shown below, click series $K_{2,1} K_{2,3}$ (the underlined lights in row 2) will switch the lights (1, 1) and (1, 3) off. After that, $K_{3,1} K_{3,3}$ in row 3 will switch the two lights in row 2 off. Finally, $K_{4,3}$ in row 4 will switch off the light in row 3.



18. [Exploration]

a) Make the start configuration given above, and switch off the first three rows by chasing.

If you want to solve the start configuration, you are almost there after chasing; except for the bottom row. For this row there is a trick, for which we will first show that it works, i.e. that you do indeed reach the null configuration.

b) Now click on light (1, 3) and chase the lights again. Surprise! The null configuration!

The obvious question now is: how do you come up with the mysterious idea to click (1,3) rather than any other light or combination of lights in the first row? Because you do not like hocus pocus, do you?

The honest answer is:

In row 1 all eight possible combinations of clicks have been tried one by one. Clicking on only (1,3) turned out to be correct. It took a bit of trying, but not too much as it is limited to trying out only the eight possible click series in the first row.

We will now explore what happened exactly in the chasing after clicking (1,3).

19. [Exploration]

Start with an empty 4×3 grid.

- a) Click on light (1,3). **Very important:** Do not use 'make a configuration', but click on (1,3) from 'execute clicks immediately'.
- b) Chase down.

Your final result is exactly the final result after chasing from the given start configuration!

In the real plan, in **question 18**, we performed the following steps, consecutively: - chasing the start configuration

- click on (1,3)
- chasing again
- ... and we were done.
- c) Explain why the 'and we were done' was to be expected.

Other lights and combinations of lights work exactly the same way. We do have to try out the eight options in the first row, though!

For the 4×3 grid there are only 8 possible click series in row 1. You have already seen the result of starting with light (1,3) in **question 18**.

20. [Exploration]

Start with a null configuration on the grid every time, and immediately begin with 'execute clicks immediately' (do NOT use 'toggle lights individually').

For the eight possible click series chase to the fourth row from an empty grid, and write down the final configurations in the table:

click series in row 1	chasing result in row 4
no clicks	
only (1, 1)	
only (1, 2)	
only (1, 3)	1 0 0
(1, 1) and (1, 2)	
(1, 1) and (1, 3)	
(1, 2) and (1, 3)	
all three lights	

21. [Exploration]

Now take a random start configuration on a 4×3 grid for which you want to find the solution (if there is one...). Call that start configuration *A*.

After chasing you reach a situation in which there can only be ones in row 4. With some luck you end up in the null configuration immediately, but that is the exception rather than the rule!

If there are still ones in row 4, you look for a click series, starting from the null configuration, that will also give the same result in row 4 after chasing as the result from chasing *A*.

So: In the table, look for the correct click series in row 1 on the empty grid that will, after chasing, yield the result of chasing *A*.

Show that this strategy of chasing and (if necessary) chasing again, does indeed work.

22. [Assignment D]

There are 2^{12} possible configurations on a 4×3 grid. Why is it, in this case, certain that for all these 2^{12} configurations the chasing result in row 4 can be found in the table from **question 20**?

E: Configurations and powers of two

This section is fairly general. You will learn nothing about solving specific situations. But you will gain insight into the whole set of possible configurations on any grid. And about how many start configurations are solvable. Key words for this part are: *Smart counting* and *clever grouping*.

Warning! This part is rather difficult and theoretical. Only work on it if you still have enough time in the morning. This part is specifically important for answering Main question D in part TWO.

23. [Exploration]

On the 4×3 grid you have a total of 12 lights. Each of these lights can be either on or off. This means that there are 2^{12} possible configurations. The 2×6 and 1×12 grids also have 2^{12} configurations. For the number of configurations only the total number of lights is important. Not the shape of the grid! We refer to this number with *N*.

What is the number of possible click series on a grid with N lights? We will also count 'click nothing' as a click series.

Classes of connected configurations

Using a toggle diagram (created through a click series) you can go from one configuration to another configuration. You already know:

- If you can get from configuration A to configuration B via toggle diagram s, then you can also go from B to A via s. In that case we call A and B connected configurations. Pay attention: A is also connected to itself, through the toggle diagram with all zeroes (i.e. do nothing)!
- If you can get from configuration A to configuration B via toggle diagram s, and from A via another diagram t to C as well, you can also go from B to C with a click series. Or, put another way: Configurations that are connected to A are also connected to each other.

24. [Exploration]

Explain: the number of configurations that is connected to configuration *A* is equal to the number of possible toggle diagrams.

Keep in mind: quite often there are fewer toggle diagrams than the number of all possible click series and thus fewer toggle diagrams than possible start configurations.

(If you do not believe this, count the number of possible configurations on a 1×2 grid and the number of possible click series and the number of toggle diagrams.)

The whole class of configurations that are connected to A is called the **configuration class of** A. In the figure on the right some connections between A and other configurations have been drawn. Of course the figure is not complete, there are probably many more. From A, for every existing toggle diagram, an arrow connects to a configuration that is connected to A. The other configurations within the class are all also connected by toggle diagrams, but that has not been drawn here. The rounded rectangle depicts the boundary of the configuration class of A.



If configuration K is not in the configuration class of A, then there is an arrow for every existing toggle diagram running from K to a configuration that is connected to K. All these configurations together form the configuration class of K.



You can continue in this manner. Eventually the whole set of configurations while be divided into configuration classes.

25. [Exploration]

Imagine: *A* and *K* are not connected, and *X* is a configuration in the configuration class of *A* and *Y* is a configuration in the configuration class of *K*. Then *X* and *Y* are not connected to each other.

a) Can you explain that?

Your explanation might, for instance, start with the observation: 'if *X* and *Y* were connected, then ... and ... would also be ...'

You can put it like this: the configuration classes of *A* and *K* are completely unconnected.

b) All solvable start configurations in one configuration class. Explain!

Of course, the following is true as well: If *B* is in the configuration class of *A*, the configuration class of *B* is exactly the same as the configuration class of *A*.

Summary

The set of all configurations is divided in configuration classes.

- All configurations in one configuration class are mutually connected;
- Configurations from different configuration classes are not connected.

26. [Exploration]

- a) If you add up the number of configurations in all classes, the result is 2^{N} . Why is that?
- b) Why are all configuration classes the same size? And what is that size (compared to the toggle diagrams)?
- c) Now explain this equation:

(Number of configuration classes) × (number of toggle diagrams) = 2^{N} .

- d) Why can the numbers for 'number of configuration classes' and for 'number of toggle diagrams' only be of the form 2^{p} with p = 0, 1, 2, 3, ...?
- e) Derive and explain (use a previous observation from the Explorations):

(Number of configuration classes) × (number of solvable configurations) = 2^{N} .

Part TWO: Own investigation

In part ONE, we explored various techniques such as *mutual toggling* (by forming groups of lights that can be mutually reached), and *chasing*. Also, several terms were introduced: *toggle diagram*, *parallel click series* and *silent click series*. These techniques and terms will prove their value when you work on the Main assignments below.

Choose wisely: Keep in mind that answering two Main assignments in depth will be appreciated more than superficially treating three or four Main assignments.

Main assignment A: 1×n grids

Consider a $1 \times n$ grid with a random start configuration. Use the Explorations from part A to find out which start configurations can or cannot be solved on a $1 \times n$ grid. Try to analyse this for all values of *n*.

Main assignment B: Chasing

'Chasing' and 'silent click series' can be used for analysing a random start configuration on a $m \times n$ grid, to see whether a given start configuration can be solved.

- a) Analyse the start configurations on a 5×5 grid (the original game) that turn out to be solvable. Make use of all possible final configurations resulting from chasing on the first row from the null configuration.
- b) Now, can you solve puzzle 3 from the 'warming up' via a targeted click series?

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

c) What can be discovered about a $m \times n$ grid with a random start configuration? Analyse the $m \times 3$ grids, for example.

Main assignment C: Everything on, everything off

It is said that on every $m \times n$ grid that is completely filled with ones, a click series can be found that leads to the null configuration. We already investigated this for $1 \times n$ grids in **question 8**.

- a) Investigate the solvability on a 2×*n* grid.
- b) Investigate a few more cases, such as an $n \times n$ grid for n = 2, 3 and 4.

Main assignment D: configuration classes and click series

This main assignments refers back to the explorations in part ONE, section E. If you skipped that part of ONE due to lack of time, there is no point in trying this main assignment!

A nice proposition:

Number of configuration classes = number of silent click series

- a) Investigate this proposition. *Hint*: Divide the click series in groups of parallel click series.
- b) Some other fun research questions:

1. The number of solvable start configurations on a m×n grid.

In that case we have $N = m \times n$. Consider only $m \ge n$. If you are chasing a given start configuration, you end up with a final configuration with just zeroes in row 1, 2, up to m -1 and possibly also ones in row m.

Two statements that you can make use of (do of course investigate their correctness):

- A start configuration and its final configuration are in the same configuration class.
- There is a maximum of 2^n different final configurations after chasing, so there is also a maximum of 2^n configuration classes.
- → Show that, in the case of a $m \times n$ grid with $m \ge n$, there are always at least $2^{(m-1) \cdot n}$ solvable configurations.

2. On a 5×5 grid only 25% of all start configurations is solvable

 \rightarrow Investigate this proposition.

Hint: Use what you found in Main assignment B and in the Explorations of part ONE, section E.